

*The Waisman Laboratory
for Brain Imaging and Behavior*



University of Wisconsin
**SCHOOL OF MEDICINE
AND PUBLIC HEALTH**

Hot Spots Conjecture and Its Application to Modeling Tubular Structures

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Abstract

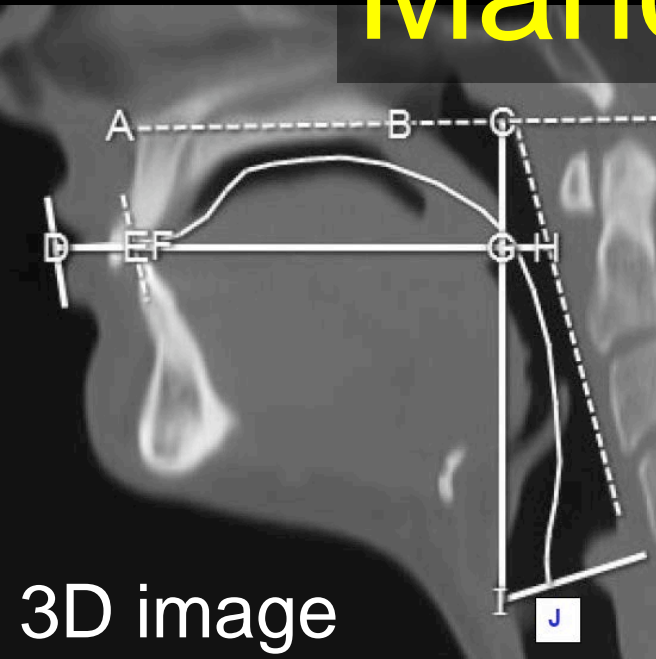
The second eigenfunction of the Laplace-Beltrami operator follows the pattern of the overall shape of an object. This geometric property is well known and used for various applications including mesh processing, feature extraction, manifold learning, spectral embedding and the minimum linear arrangement problem. Surprisingly, this geometric property has not been mathematically formulated yet. This problem is directly related to the somewhat obscure hot spots conjecture in differential geometry. The aim of the talk is to raise the awareness of this nontrivial issue and formulate the problem more concretely. As an application, we show how the second eigenfunction alone can be used for shape modeling of tubular structures such as the human mandible obtained from CT images and determining the diameter of biological graphs and networks.

Talk is based on the following paper:
Chung, M.K., Seo, S., Adluru, N., Vorperian, H.K. 2011. Hot spots conjecture and its application to modeling tubular structures, 2nd International workshop on machine learning in medical imaging (MLMI), MICCAI workshop.

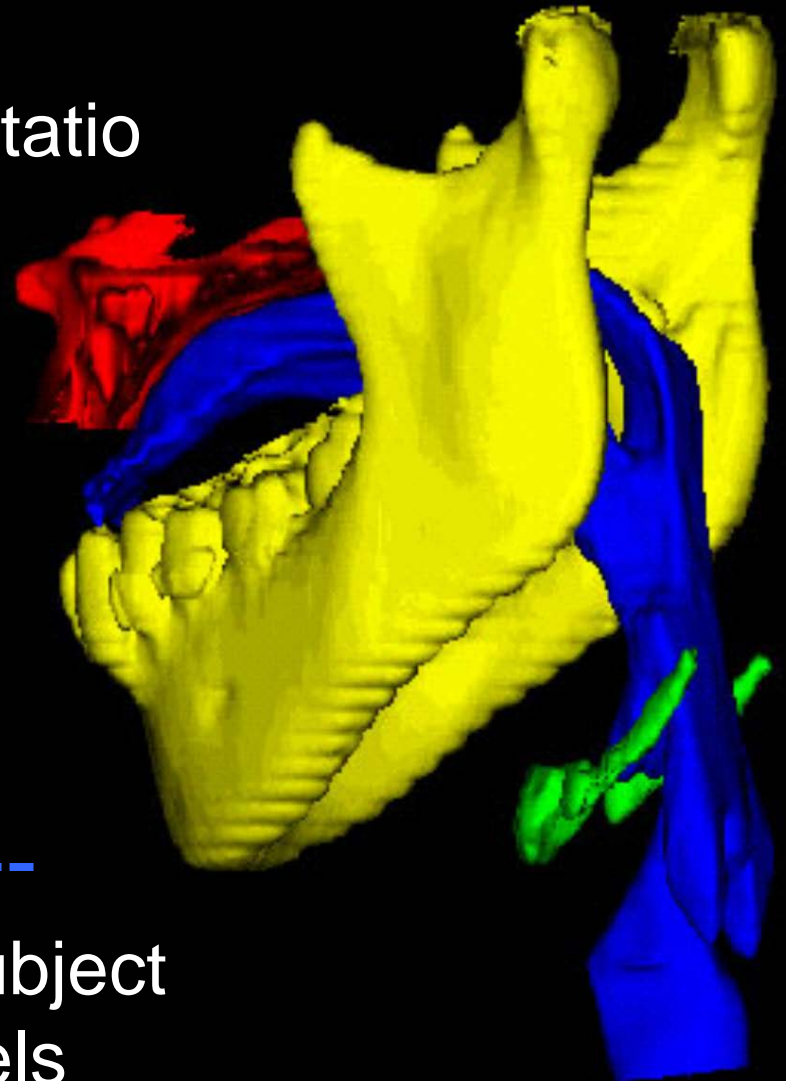
<http://www.stat.wisc.edu/~mchung/papers/chung.2011.MLMI.pdf>

MATLAB *code:*
<http://brainimaging.waisman.wisc.edu/~chung/lb>

Mandible surface model

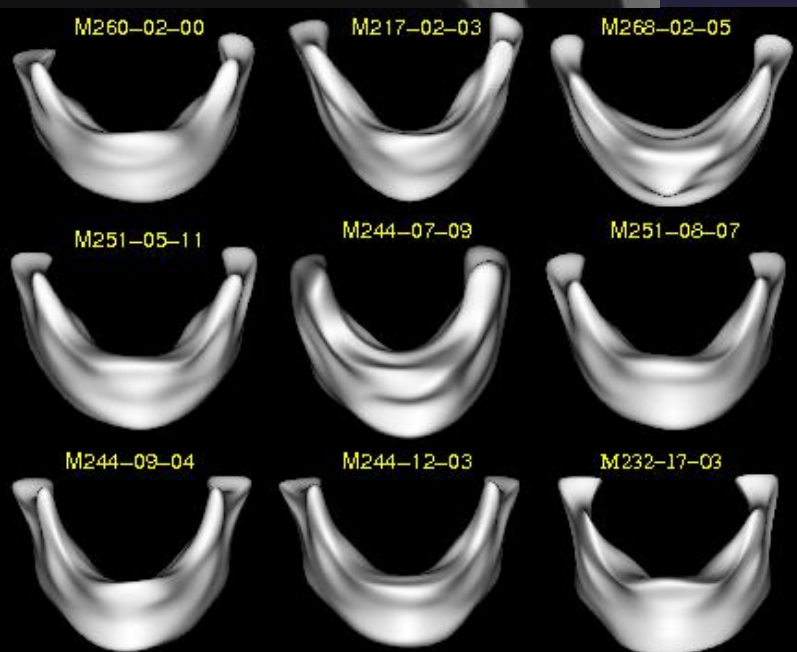


segmentatio

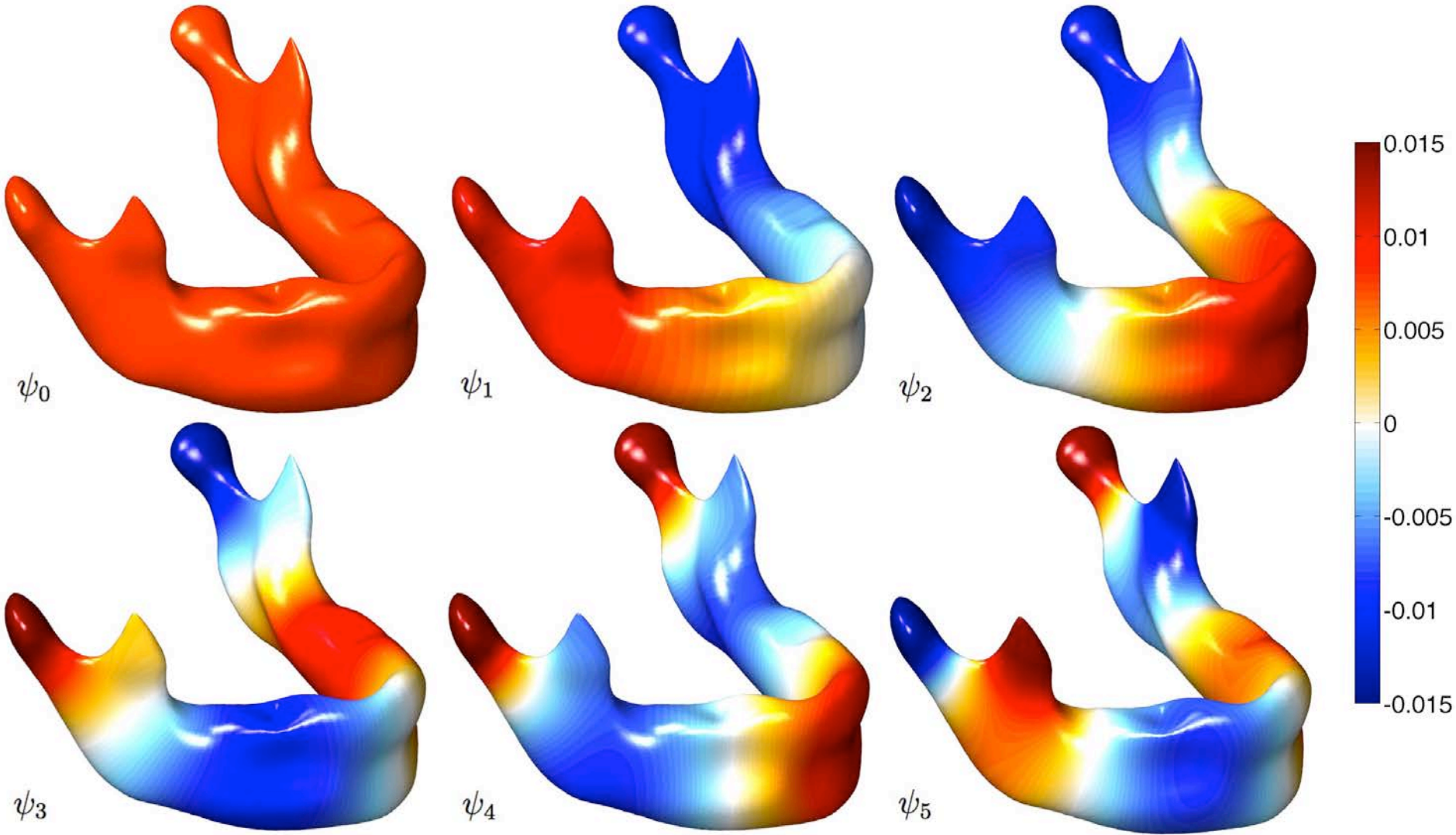


70 subject models

3D image



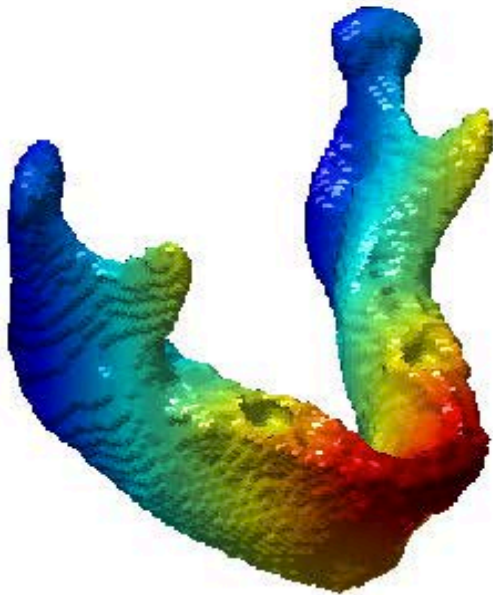
Laplace-Beltrami eigenfunctions on mandible



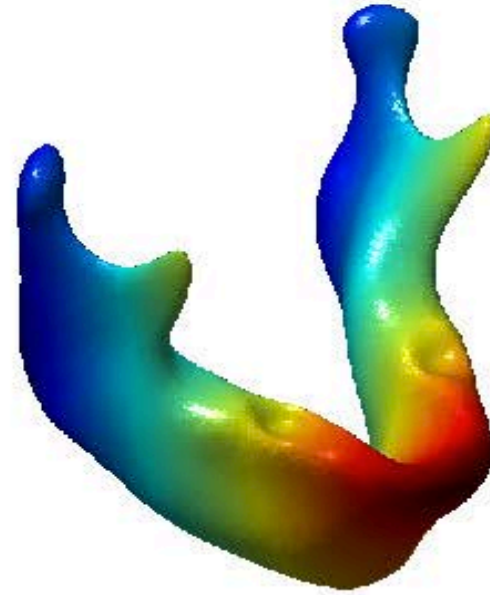
Heat kernel smoothing on manifold

Heat kernel:
$$K_t(p, q) = \sum_{i=0}^{\infty} e^{-\lambda_i t} \psi_i(p) \psi_i(q)$$

$$K_t * f = \int_{\mathcal{M}} K_t(p, q) f(q) dq$$



X-coordinate on
mandible
surface

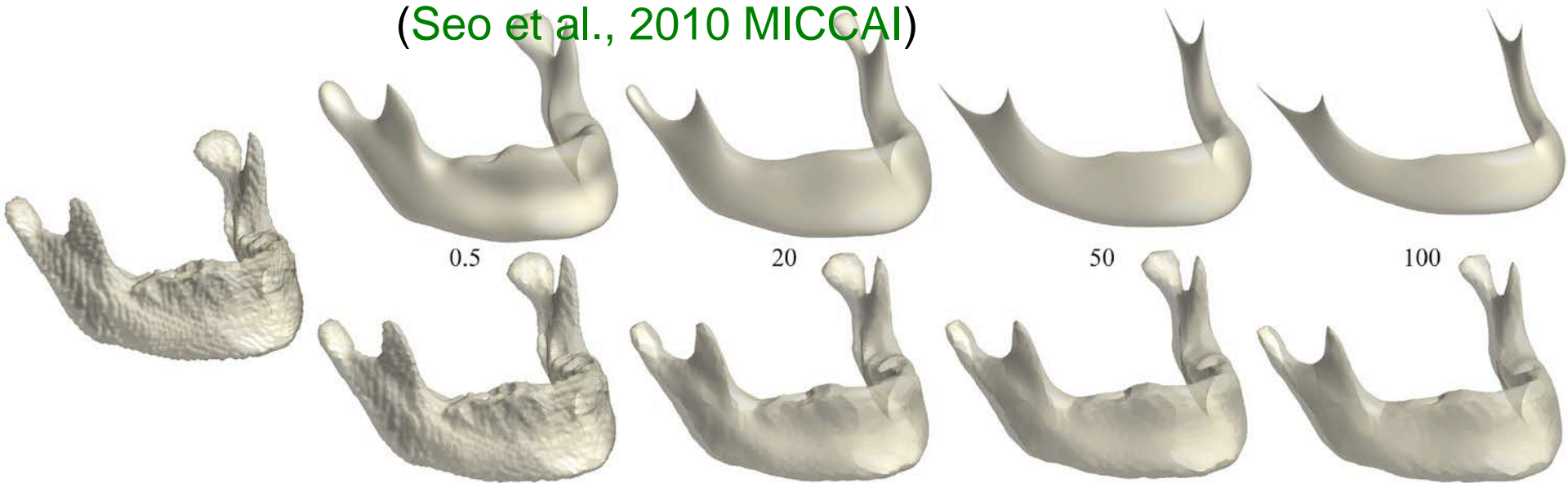


smoothed with bandwidth 10
and 1269 eigenfunctions

Heat kernel smoothing on mandible shape

Heat kernel smoothing based on LB-eigenfunctions

(Seo et al., 2010 MICCAI)

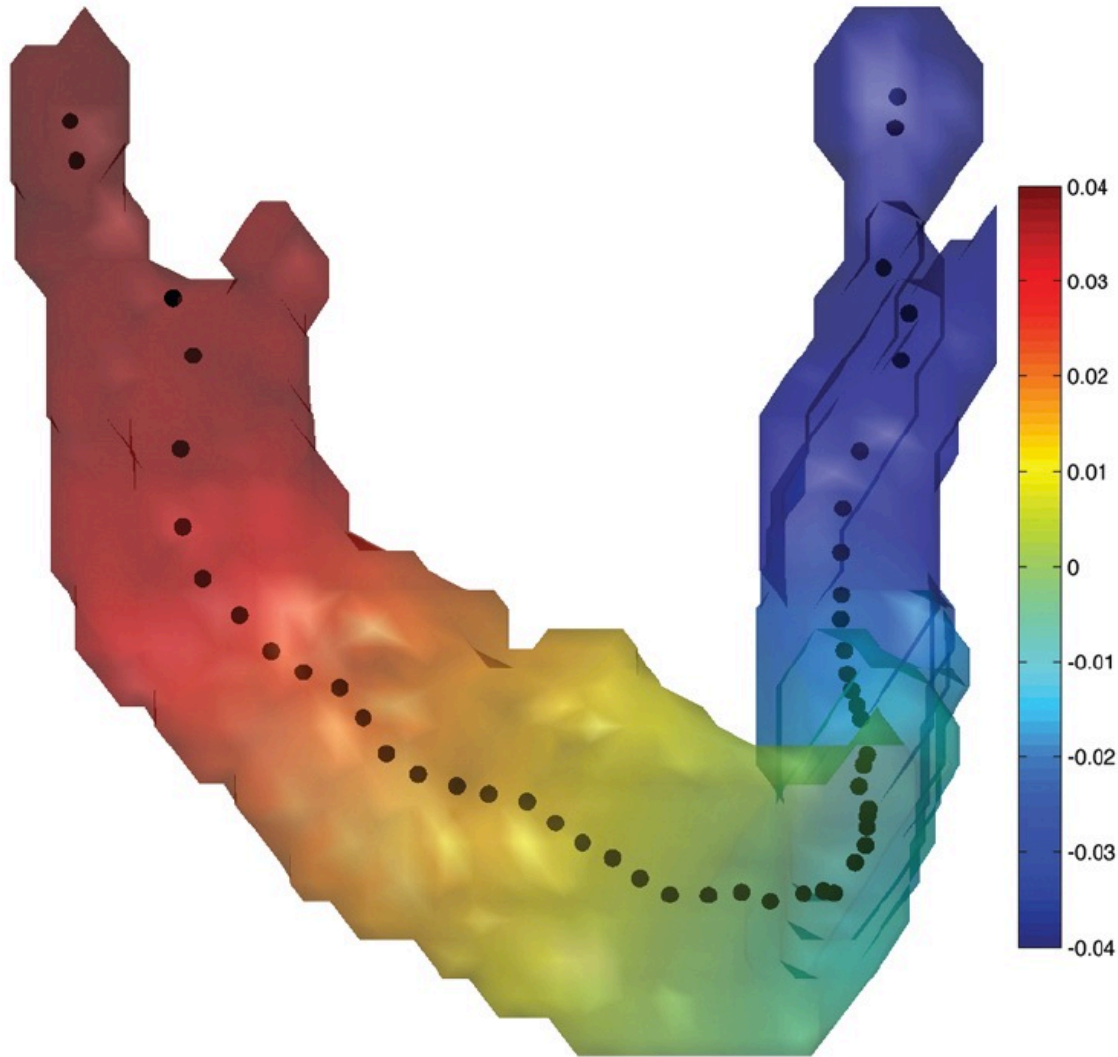


Heat kernel smoothing based on linearization of kernel
(Chung et al., 2005 NeuroImage, IPMI)

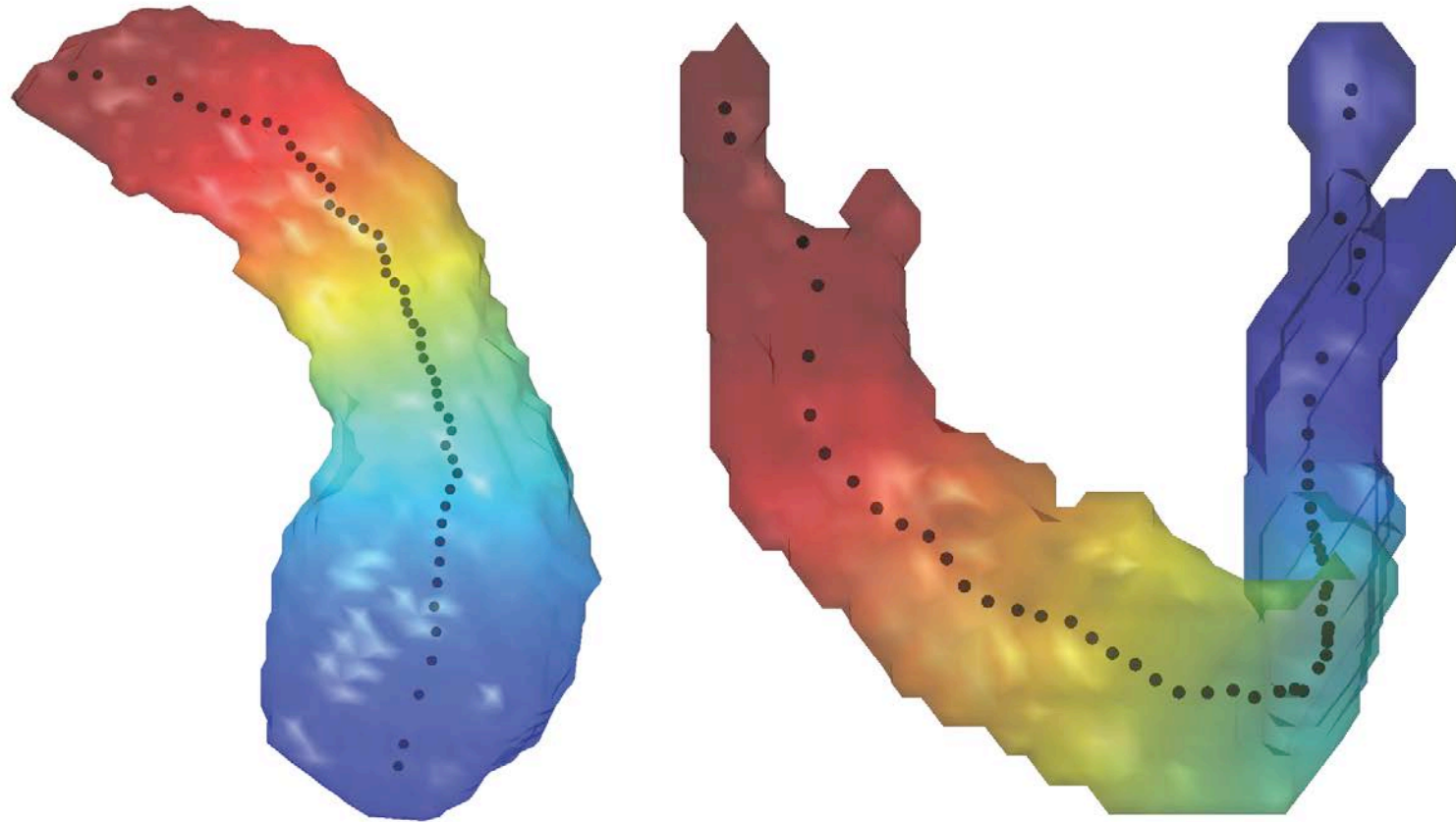


FreeSurfer, AFNI, MNI-pipeline, SurfStat

Second eigenfunction of mandible

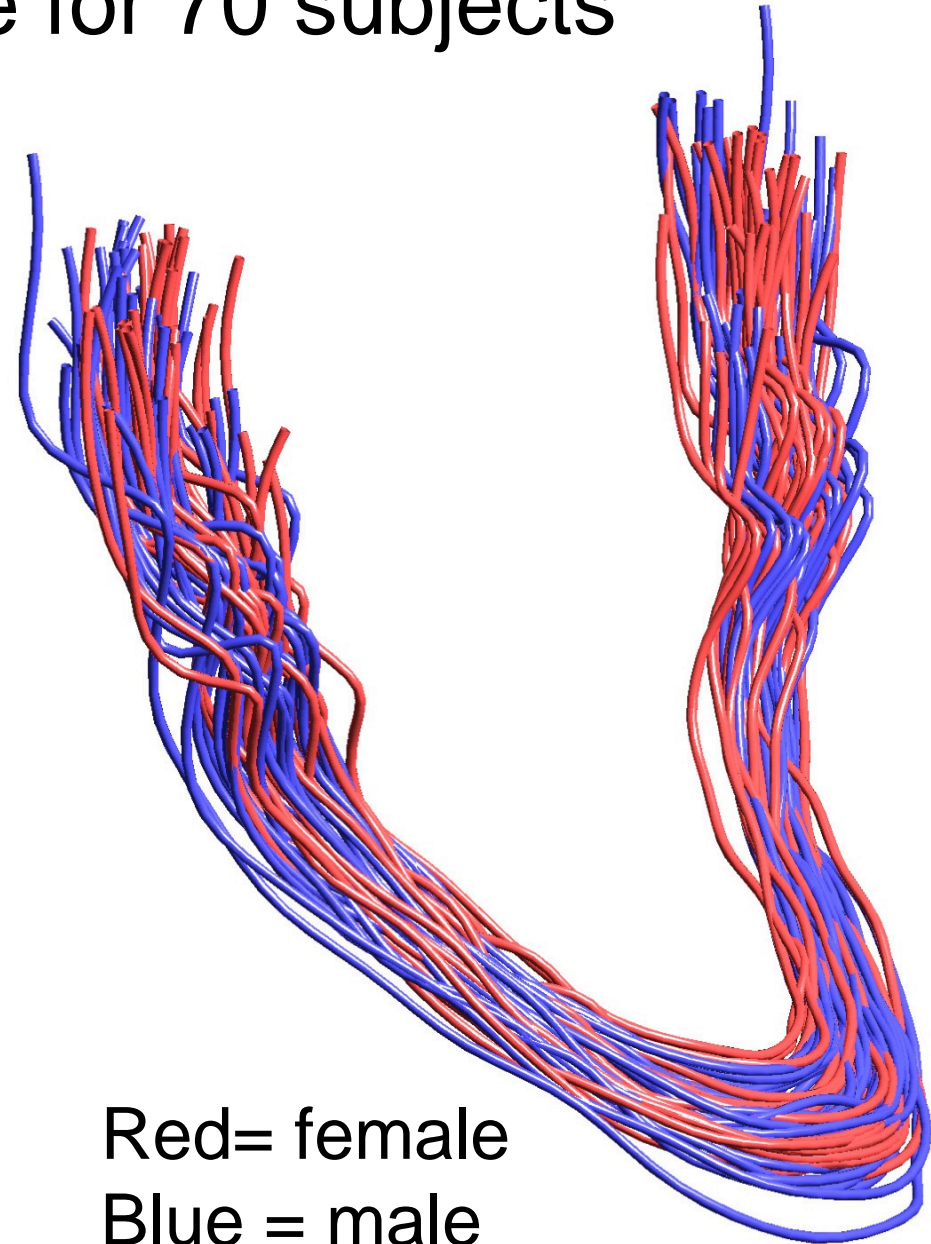
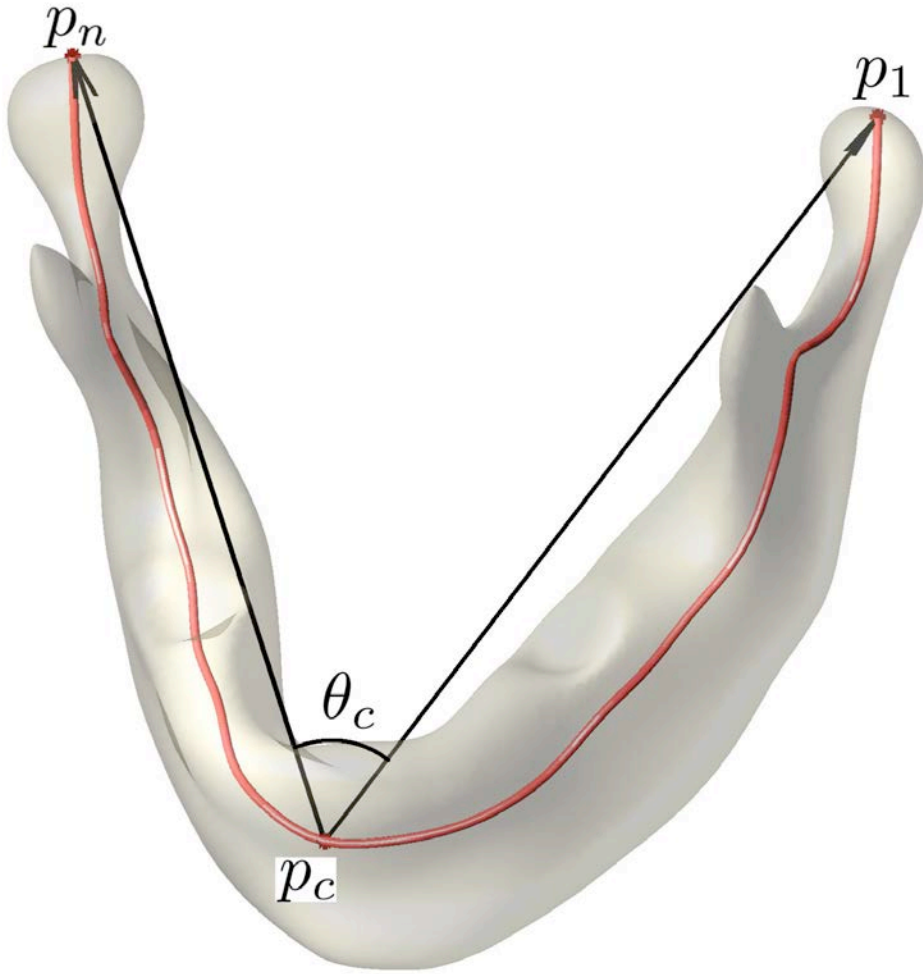


Second eigenfunction of elongated objects



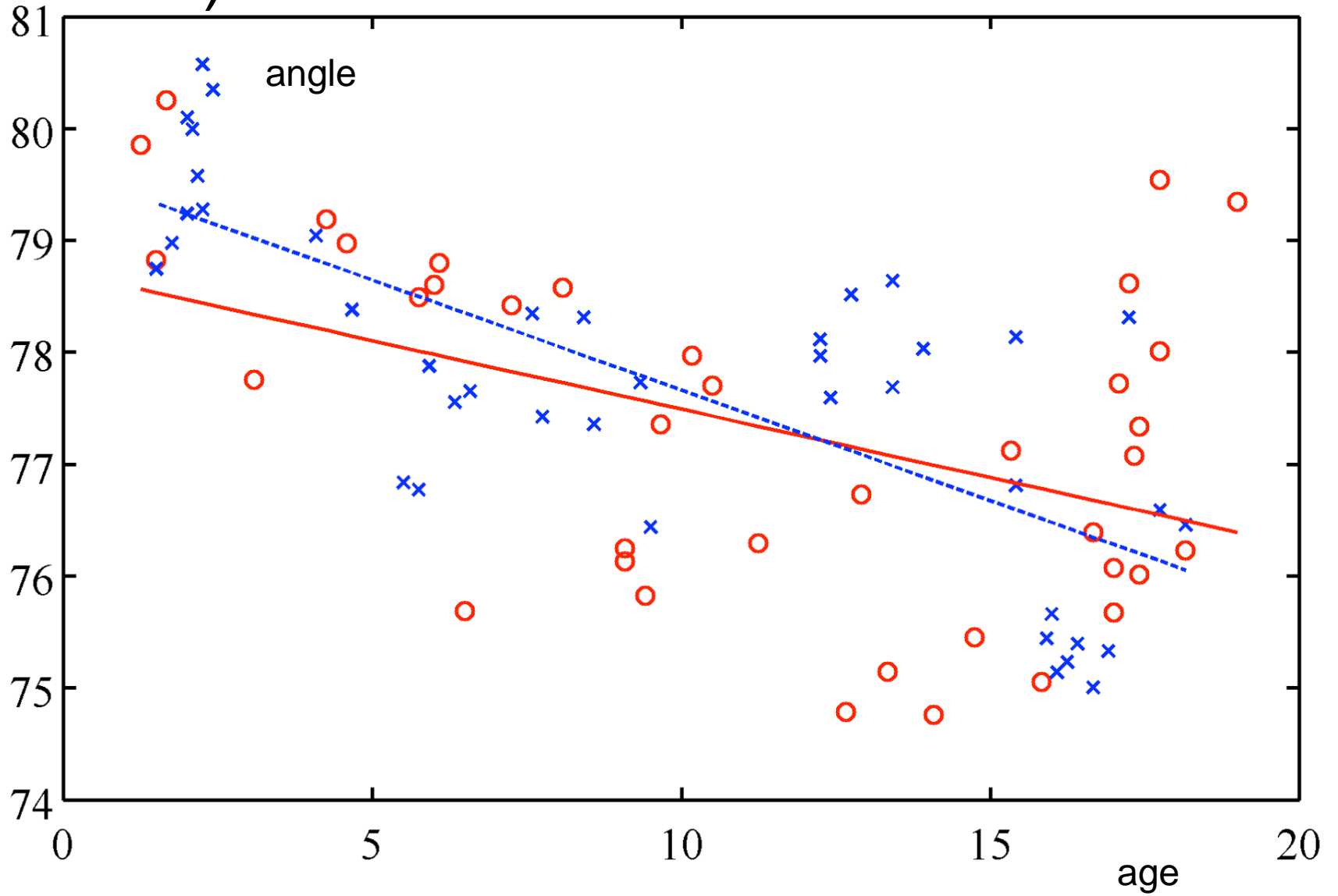
Dots are the centroids of the contour bands

Centerline of mandible for 70 subjects

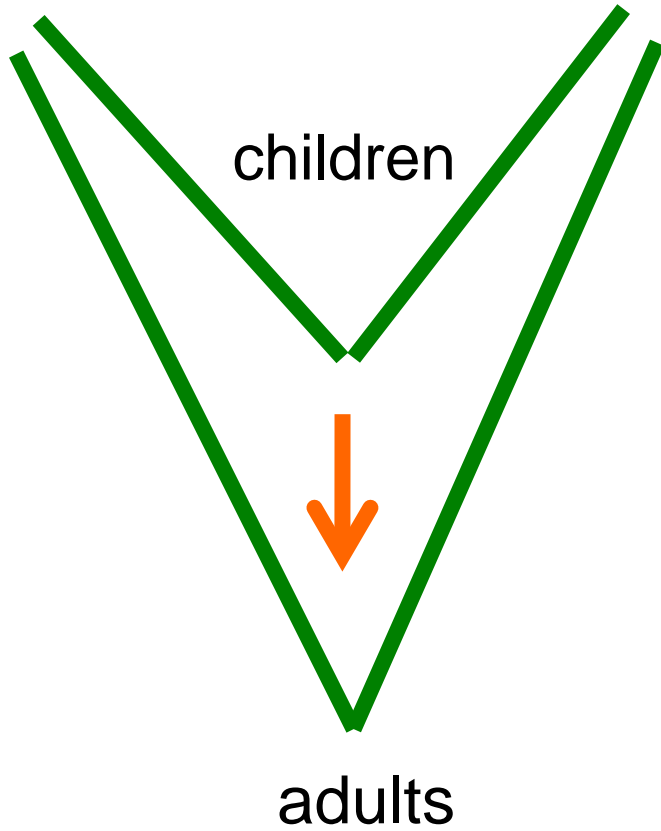


Red= female
Blue = male

Elongation of mandible (angle decrease)



Our biological findings



←
More elongated
growth for male

